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THESIS

EQUILIBRIUM ANALYSIS OF EFFECTS
OF A PRICE CHANGE OF AN INPUT FACTOR
IN THE CONTEXT OF INPUT-OUTPUT SYSTEM

by

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Equilibrium Analysis of Effects of a Price Change of an Input Factor in the Context of Input-Output System

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ABSTRACT

This paper is an attempt to model the effects of price change of a primary input factor into a segment of an economy. The primary input factor referred to is petroleum and the segment of the economy, the energy sectors. Labor is considered as another primary input factor.

Market equilibrium is assumed to be stable and the disturbance caused by a price change in a primary input factor results in a new equilibrium state. Three approaches are made to define or specify this new state of equilibrium. Input-output economics is the primary basis of all the three approaches. Having analyzed and defined the new equilibrium state gave results that could serve as bases in making policy measures relative to the nature of the disturbance.

TABLE OF CONTENTS

I. INTRODUCTION

An economy or a segment of an economy is said to be in a state of equilibrium if all goods and services available for consumption are cleared at the prevailing market prices and that all productive factors that are offered at the prevailing market prices are fully employed. Such an equilibrium may be stable or unstable. A disturbance is encountered when a change in price of the consumption goods or of the productive factors arise. Quantities demanded change and the market adjustment mechanism is assumed to work until a new equilibirum is reached.

This study attempts to formulate alternative approaches in defining or specifying a new equilibirum state as a result of a disturbance – a change in price of one input factor of production. The study assumes stability of equilibrium, i.e., a disturbance results in a return to equilibrium. It is envisioned that being able to define this new equilibirum state could give some meaningful bases in making conclusions and policy measures relative to the nature of the disturbance.

The industries or sectors referred to are the energy sectors, or those industries which use energy-generating materials as a primary input. Labor is also assumed to be another primary input factor of production. The energy-generating material referred to is crude oil or petroleum

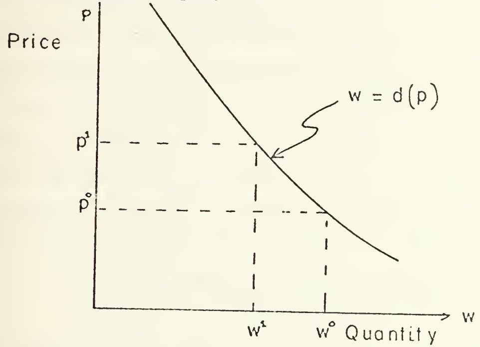
and that substitutes available are assumed to be negligible (this could be particularly true for non-oil producing countries). No attempt is made to state explicitly these sectors. It is assumed that input-output matrices of coefficients for both primary and intermediate inputs are available. However, this study is more hypothetical than real. The references to petroleum and labor provide motivation in the possible application of the analysis.

II. THE NATURE OF THE PROBLEM

A. CONSEQUENCES OF CHANGES IN PRICE OF AN INPUT FACTOR

1. Change in Quantity Demanded

A change in price of one input factor will change the quantity demanded. Figure 1 below shows a simple demand curve. If w represents the quantity of goods demanded and p its price, the function $w = d(p)$ is generally assumed to be downward-sloping.



A Demand Curve

Figure 1

Suppose the present unit price of w is p^o, then the quantity demanded is w^o. Increasing p to p^1 decreases w^o to w^1. The change in the quantity demanded $\Delta w = w^o - w^1$ represents a decrease in the total quantity of w which is an input factor.

2. Change in Quantity Demanded of the Other Factor

Suppose there are two factors of production such that for producers to produce q as output, they need w_1 and w_2 as their primary inputs, i.e.,

$$q = f(w_1,w_2) \quad ,$$

where f is a function which describes the relationship of how the inputs w_1 and w_2 are related to q. The function f is called a production function. In the economic theory of production, it is asserted that there are a number of input combinations available to producers to provide a given level of output. It is assumed though that producers always use the "best mix" of inputs. Thus, a change in one input factor is expected to generate a change in the other. This will be further discussed in Sec. III below.

3. Change in Levels of Output and Price

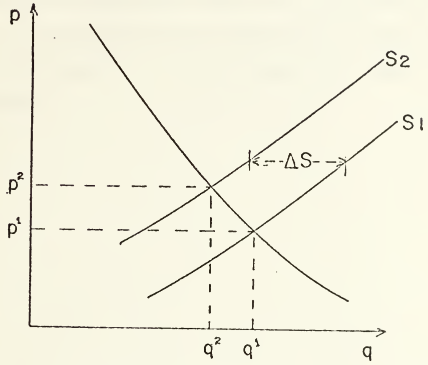
Decreasing the amount of input factors results in a decrease in the output of an industry. The higher costs of inputs results in higher cost of production and producers will increase price.

4. Change in Quantity Demanded by Consumers

Assuming that the demand functions of consumers do not change, the reduced output of goods and change in prices affect the quantities demanded by consumers which in turn affect the level of production. Consumers' demand functions may also change, however, the above-stated assumption will be maintained throughout the study to simplify the analyses.

B. THE PROBLEM DEFINED

Figure 2 below illustrates the nature of the problem.



Supply Demand Curve for One Indusiry

Figure 2

In this study it is assumed that each industry produces only one commodity. Let S_1 be the present total output of an industry available for consumption and let q^1 be the present total demand from that industry. At the price p^1, supply equals demand and an equilibrium exists. Now, suppose a disturbance occurs such that the supply of good q decreases from S_1 to S_2. Associated with this change is an increase

in the price of q to p^2. A change in quantity demanded from q^1 to q^2 occurs which is Δq. Suppose $\Delta q = \Delta S$ (ΔS = change in supply) exists, how will this state of equilibrium be specified or defined for n sectors or industries?

The following three approaches are designed to define this state of equilibrium under some restrictive assumptions which will be spelled out in the course of the analysis. Input-output analysis is the primary tool used in all the approaches.

III. APPROACHES TO THE PROBLEM

A. STATIC INPUT-OUTPUT ANALYSIS AND ASSUMED DEMAND ELASTICITIES

The first approach to define a new state of equilibrium due to a disturbance uses static input-output analysis. The changes in the levels of outputs for final demand and the corresponding changes in prices of goods are calculated for each sector. These changes are assumed to be generated by changes in the quantities of primary input factors which are previously caused by a change in price of one of these input factors. Static input-output analysis assumes no production lag and a long run equilibrium. This analysis will attempt to specify or define this new state of equilibrium. To do this, at constant level of income, consumers' elasticities of demand for each good produced by each industry are assumed to be known and that their cross elasticities are assumed to be identically zero. Since, a change in price of a factor input results in a change in prices of the output goods, this is assumed to cause a decrease in quantity demanded by consumers. At the new equilibrium point, this change in quantity demanded by consumers for good i equals the change in level of output for final demand of good i. This is the underlying basis of this method in defining a new state of equilibrium.

B. STATIC INPUT-OUTPUT ANALYSIS AND AN AGGREGATE PRODUCTION FUNCTION

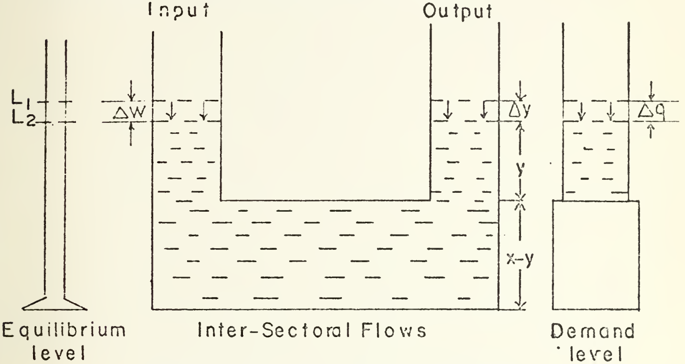
The second approach to define a new equilibrium state uses also the static input-output model. However, this differs from the first in the sense that it assumes an aggregate production function that explains the relationship of the total inputs of all sectors to the total outputs for final demand. The Cobb-Douglas production function is shown to satisfy the requirements of input-output economics and is therefore a valid function. The outputs are assumed to be expressable in the same units and an average price of output is calculated based on the profit-maximizing behavior of producers. From this average price, the prices of each good produced by each sector are estimated using the input-output matrices. Again at constant level of income, elasticities of consumers demand of each good are assumed to be known with all cross elasticities equal zero. At equilibrium, the total change in output due to the price increase in a factor input equals the total change in consumers' demand due to price changes of goods. From this point, the analysis is similar to the first approach in specifying the new state of equilibrium.

C. THE WALRAS-LEONTIEF GENERAL EQUILIBRIUM SYSTEM

This third and last approach to define an equilibrium state given a disturbance is included in this study to illustrate the complexity of attempting to calculate equilibrium solution

taking into account the consumer demand for goods, the price-cost equations, the input-output equations, and the demand-supply equations for primary input factors. The functions used in the second approach are verified to yield equilibrium solutions.

Since, the above three approaches use input-output analysis, a flow diagram of inputs and outputs is drawn to give a picture of the basis of the equilibrium analysis.



Input-Output Equilibrium System

Figure 3

From the above diagram, the problem being studied is that given a disturbance which lowers the level of input from L_1 to L_2, a corresponding change occurs in the level of output. At equilibrium, the level of consumer demand should be also at L_2. Before proceeding to the various approaches, one comment is made regarding the direction of the chain of effects. This analysis takes the disturbance to occur at the left-hand

side. However, consumers' demand also affect the level of output and hence the level of input. In the succeeding analysis attempts are made to cover this interdependence.

IV. THE FIRST APPROACH: STATIC INPUT-OUTPUT ANALYSIS AND ASSUMED DEMAND ELASTICITIES

A. AN INTRODUCTION TO INPUT-OUTPUT ECONOMICS

In input-output analysis, each industry is assumed to have only one output good or commodity. Thus if there are n industries there are also n goods. Each industry uses two basic types of inputs, namely; the intermediate inputs and primary inputs. Intermediate inputs are those products of industries used as inputs by other industries. This includes the portion of output of an industry which it uses to produce its output. Primary inputs are exogenous factors of production generally classified as labor. For this analysis, petroleum is classified as primary input factor into the energy sectors or industries. The following discussion in input-output economics is aimed towards developing the equations that could give the change in the output available for final demand given a change in the quantity of a primary input.

1. The Input-Output Accounting Matrix

Let x_i be the total output of industry i and x_{ij} the amount of output of industry i used as input by industry j. Let y_i be the net output of industry i. Then the over-all input-output balance equation of the whole system is:

$$(1) \qquad x_i = \sum_{j=1}^{n} x_{ij} + y_i \quad , \quad i=1,\ldots,n$$

Also, let w_{hj} be the amount of the $h^{\underline{th}}$ primary input factor to industry j. Then,

$$(2) \qquad w_h = \sum_{j=1}^{n} w_{hj} \quad , \ h=1,\ldots,k,$$

where k is the total number of primary factors of production. The above equations can be shown in table form.

		Intermedicte Output to					Final Demand	Gross Output
		1	2	.	.	n		
Intermediate Input From	1	x_{11}	x_{12}	.	.	x_{1n}	y_1	x_1
	2	x_{21}	x_{22}	.	.	x_{2n}	y_2	x_2
	$\vdots$	$\vdots$	$\vdots$			$\vdots$	$\vdots$	$\vdots$
	n	x_{n1}	x_{n2}	.	.	x_{nn}	y_n	x_n
Primary Inputs	1	w_{11}	w_{12}	.	.	w_{1n}		
	2	w_{21}	w_{22}	.	.	w_{2n}		
	$\vdots$	$\vdots$	$\vdots$			$\vdots$		
	k	w_{n1}	w_{n2}	.	.	w_{nn}		

Input-Output Accounting Matrix

Figure 4

2. <u>Production Functions</u>

In input-output economics, a fixed-proportion production function is assumed to explain the relationship between inputs and outputs. This function has the form,

$$(3) \qquad q = c[\min(w_1/a, w_2/b)]$$

where c is a constant indicating the productivity of the whole industry and a and b are constants showing the way w_1 and w_2 should be mixed. Thus a units of w_1 and b units of w_2 are needed to produce c units of q. Input-output economics assumes that every industry uses the "best mix" [1]. Thus in general,

(4) $$\text{output} = \text{input}/\text{output}$$

To produce n times of the present output q^o, it is necessary to have n times the present input x^o, i.e.,

$$nq^o = nx^o/\text{constant}$$

This shows that the production function in input-output economics has constant return to scale and that w_1 and w_2 cannot be substituted for each other. Thus

(5) $a_{ij} = x_{ij}/x_j$ and (6) $b_{hj} = w_{hj}/x_j$,

where a_{ij} and b_{ij} are constants and are called the intermediate and primary input coefficients respectively. The matrices of input coefficients can therefore be formed.

$$
\begin{array}{c|cccc}
 & 1 & 2 & \cdots & n \\
\hline
1 & a_{11} & a_{12} & \cdots & a_{1n} \\
2 & a_{21} & a_{22} & \cdots & a_{2n} \\
\vdots & \vdots & \vdots & & \vdots \\
n & a_{n1} & a_{n2} & \cdots & a_{nn} \\
\hline
1 & b_{11} & b_{12} & \cdots & b_{1n} \\
2 & b_{21} & b_{22} & \cdots & b_{2n} \\
\vdots & \vdots & \vdots & & \vdots \\
k & b_{k1} & b_{k2} & \cdots & b_{kn}
\end{array}
$$

Define the matrices A and B such that,

$$
A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \quad \text{and } B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ b_{k1} & b_{k2} & \cdots & b_{kn} \end{bmatrix}
$$

A and B are the matrices of intermediate and primary input coefficients respectively.

3. The Price Equations

Let $k = 2$, i.e., there are two primary input factors of production and let r_1, r_2 be their unit prices. Let p_i, $i=1,\ldots,n$ be the unit prices of the $i^{\underline{th}}$ good produced

by the $i^{\underline{th}}$ sector. Since long run equilibrium is assumed, profit equals zero and the cost of a unit of good equals its price. Thus for the $i^{\underline{th}}$ industry,

$$(7) \qquad \sum_{j=1}^{n} p_i x_{ij} + r_1 w_{1j} + r_2 w_{2j} = p_j x_j$$

Using equations (5) and (6), the above equation becomes:

$$\sum_{j=1}^{n} p_i a_{ij} x_j + r_1 b_{ij} x_j + r_2 k_{2j} x_j = p_j x_j$$

x_j cancels out and,

$$\sum_{j=1}^{n} p_i a_{ij} + r_1 b_{1j} + r_2 b_{2j} = p_j$$

In matrix notation,

(7a) $\quad P'A + r_1 b_1' + r_2 b_2' = p'$, where P' and b_h',

$h=1,2$ are transposes of the column vecotrs P and b_h respectively.

4. Gross Output Needed to Produce a Given Final Demand

Substituting equation (5) in equation (1) gives,

$$(8) \qquad \sum_{j=1}^{n} a_{ij} x_j + y_i = x_i$$

Define the vector of gross outputs and final outputs as:

$$X = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \quad \text{and} \quad Y = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \quad \text{respectively.}$$

In matrix notation equation (8) becomes,

$$AX + Y = X \quad , \text{ and solving for X gives,}$$

(9) $$X = (I-A)^{-1}Y$$

where $(I-A)^{-1}$ is the inverse of the matrix $(I-A)$, I being an nxn identity matrix.

5. <u>Feasible Output</u>

The output of the economy or a segment of the economy is restricted by the available primary factor [2]. Writing equation (6) in matrix notation gives,

(10) $$W = BX \quad ,$$

where W is the vector of primary inputs. Substituting equation (9) in equation (10) results in:

(11) $$W = B(I-A)^{-1}Y \quad .$$

This equation is the relationship which will be extensively used hereafter. It can be seen that the final output vector Y is dependent upon the input vector W.

6. Open and Closed Input-Output Systems

The model described above is an open system since the final demand sector is assumed to be exogenous. A system is said to be closed if the consumer sector is considered as another industry, its input being the output for final demand and its output labor. In the present analysis, if petroleum were an import product, as it is the case for non-oil countries, it can be regarded as another primary input factor. The reference to static input-output analysis in this paper is interpreted to refer to open static input-output system.

B. CHANGE IN PRICE OF ONE INPUT FACTOR

Assume without loss of generality that there are three energy sectors whose primary inputs are w_1 and w_2. Let the price of w_1 be increased by Δr_1. Assume that the demand for w_1 is a function of its price and is given by:

$$w_1 = be^{-a \log r_1} \tag{12}$$

and that the substitute for w_1 is negligible. Define ε_1 as the elasticity of demand for w_1 such that,

$$\varepsilon_1 = (r_1/w_1)(dw_1/dr_1) \tag{13}$$

Solving for ε_1,*

$$\varepsilon_1 = [r_1/w_1][be^{-a \log r_1}][d/dr_1(-a \log r_1)]$$

$$= [r_1/w_1][(-ab/r_1)(e^{-a \log r_1})]$$

$$= -a$$

Let w_1 refer to petroleum and let w_2 refer to labor. A change Δr_1 will result in a change in w_1 demanded. Thus,

$$(\Delta w_1/w_1)/(\Delta r_1/r_1) = a \quad , \text{ or}$$

(13a) $$\Delta w_1 = a\ w_1\ \Delta r_1/r_1$$

If Δr_1 is positive then $w_1 < 0$.

Since the industries use a fixed proportion production function, a decrease in w_1 will mean an excess of w_2. This can be illustrated by Figure 5 below.

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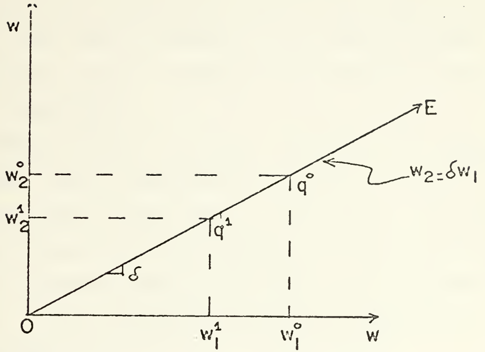
*In economics, the negative sign for demand elasticity is usually understood and therefore not used. Thus, $\varepsilon_1 = a$.



Fixed Proportion of Input Factors

Figure 5

The proportion of w_1 and w_2 is fixed and lies along the line OE which is called the expansion path. If q^o were the present level of output at w_1^o and w_2^o quantities of inputs, then a decrease of $\Delta w_1 = w_1^o - w_1^1$ results in an excess of $\Delta w_2 = w_2^o - w_2^o$ since w_1 and w_2 cannot be substituted for each other and the fixed proportion of δ has to be maintained. In other words, the optimal mix of w_1 and w_2 is only along line OE. It is assumed that the cost of labor r_2 remains constant and that the change in labor is absorbed by the other segment of the economy.

C. CHANGE IN OUTPUT FOR FINAL DEMAND

Equation (11) implies that only a feasible amount of output is produced if the primary input is limited. The decrease in the amounts of w_1 and w_2 will cause a reduction in the total output and hence in the output available for final demand. This is based on the assumption that equation (11) is satisfied.

Define $F \equiv B(I-A)^{-1}$ and f_{hj} to be the h-j$\underline{\text{th}}$ element of the F matrix. Then F indicates the total requirements of primary input per unit of final good.* For instance, the first column of F gives the total amount of primary factors 1 to k needed to produce one unit of output for final demand by sector 1. For two primary factors, the total amount of w_1 and w_2 are:

$$w_1 = b_1' \, (I-A)^{-1}Y \quad , \quad \text{and}$$

$$w_2 = b_2' \, (I-A)^{-1}Y \quad \text{using equation (11)} \; .$$

For changes in input factor, similar equations can be derived. Thus,

$$\Delta w_1 = b_1' \, (I-A)^{-1}Y \quad , \quad \text{and}$$

$$\Delta w_2 = b_2' \, (I-A)^{-1}Y \quad .$$

---

*Reference [1], p. 77.

If the number of factor inputs equal the number of sectors $(k=n)$, then the equation $\Delta W = B(I-A)^{-1}Y$ has a unique solution, assuming that B is nonsingular. However if $k < n$, there will be more unknowns than the number of equations. For the special case k=2 and n=3,

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} f_{11}y_1 + f_{12}y_2 + f_{13}y_3 \\ f_{21}y_1 + f_{22}y_2 + f_{23}y_3 \end{bmatrix}$$

Assume that y_3 is given from the outside (y_1 or y_2 may also be assumed and in the kxn case, n-k elements of the final demand vector had to be assumed),

$$\begin{bmatrix} w_1 - f_{13}y_3 \\ w_2 - f_{23}y_3 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} .$$

Solving for y_1 and y_2,

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}^{-1} \begin{bmatrix} w_1 - f_{13}y_3 \\ w_2 - f_{23}y_3 \end{bmatrix}$$

or,

(14) $$\underline{Y} = \underline{F}^{-1} \begin{bmatrix} w_1 - f_{13}y_3 \\ w_2 - f_{23}y_3 \end{bmatrix}$$

where $\underline{Y}$ is the final demand vector less the third element y_3, and $\underline{F}^{-1}$ is the inverse of the F matrix, with its third column corresponding to the assumed element of the final demand vector, removed. Thus, a change Δw_1 and Δw_2 will cause a change Δy_1, Δy_2 and Δy_3. Since Δy_3 has been fixed, Δy_1 and Δy_2 are:

$$(14a) \quad \begin{bmatrix} \Delta y_1 \\ \\ \Delta y_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} \Delta w_1 \\ \\ \Delta w_2 \end{bmatrix} = \begin{bmatrix} g_{11}\Delta w_1 + g_{12}\Delta w_2 \\ \\ g_{21}\Delta w_1 + g_{22}\Delta w_2 \end{bmatrix} ,$$

where g_{hj}, $h = 1,2$ and $j=1,\ldots,3$ are the elements of the $\underline{F}^{-1}$ matrix.

D. CHANGES IN THE VECTOR OF PRICES

Now, recall the price equation (7a) which after solving for P,

$$P' = (r_1 b_1' + r_2 b_2')(I-A)^{-1} \quad .$$

Expanding gives,

$$(p_1, p_2, \ldots, p_n) = [\sum_{j=1}^{n} (r_1 b_{1j} + r_2 b_{2j}) c_{j1}, \cdots, \sum_{j=1}^{n} (r_1 b_{1j} + r_2 b_{2j}) c_{jn}] \ .$$

A change Δr_1 in the price of the first primary input factor will result in a change in the equilibrium price vector P. Thus,

$$P+\Delta P = [(r_1+\Delta r_1)b_1' + r_2b_2'](I-A)^{-1}$$

and solving for ΔP,

$$\Delta P = \Delta r_1 b_1'(I-A)^{-1} \quad . \tag{15}$$

It can be seen that if Δr_1 is positive, ΔP is also positive.

E. DEMAND AND INCOME ELASTICITIES

Let q_i be the quantity demanded for the $i^{\underline{th}}$ good in an n-good economy such that,

$$q_i = h(p_1, p_2, \ldots, p_n, I) \quad , \tag{16}$$

where p_i is the price of the $i^{\underline{th}}$ good, I is the total level of income, and h is the function that relates q_i to prices and income. Taking the total differential of equation (16)

$$dq_i = \sum_{j=1}^{n} \frac{\partial q_i}{\partial p_j} dp_j + \frac{\partial q_i}{\partial I} dI \quad , \quad j=1,\ldots,n \; .$$

Dividing through by q_i and multiplying each element of the first term on the right by p_j/p_j and the second term by I/I,

$$dq_i/q_i = \sum_{j=1}^{n} \left(\frac{\partial q_i/q_i}{\partial p_j/p_j}\right)(dp_j/p_j) + \left(\frac{\partial q_i/q_i}{\partial Y/Y}\right)(dI/I) \quad .$$

But $\frac{\partial q_i/q_i}{\partial p_j/p_j} = \varepsilon_{ij}$ which is the elasticity of demand of the $i^{\underline{th}}$ good with respect to the $j^{\underline{th}}$ price, and $\frac{\partial q_i/q_i}{\partial I/I} = n_i$ which is by definition the income elasticity of the $i^{\underline{th}}$ good. Thus the above equation becomes,

$$(17) \qquad dq_i/q_i = \sum_{j=1}^{n} \varepsilon_{ij}\, dp_j/p_j + n_i\, dI/I \quad .$$

Increasing all the prices in the same proportion as Y, i.e., $dp_j/p_j = dI/I$ for all j, then the relative change in quantity demanded is zero. Or, $dq_i/q_i = 0$. Hence equation (17) in expanded form is:

$$- \begin{bmatrix} \varepsilon_{11}dp_1/p_1 + \ldots + \varepsilon_{1n}dp_n/p_n \\ \vdots \qquad\qquad \vdots \\ \varepsilon_{n1}dp_1/p_1 + \ldots + \varepsilon_{nn}dp_n/p_n \end{bmatrix} = \begin{bmatrix} n_1 dI/I \\ \vdots \\ n_n dI/I \end{bmatrix} \quad .$$

or,

$$(\varepsilon_{i1} + \varepsilon_{i2} + \ldots + \varepsilon_{in}) = n_i \qquad , \quad i=1,\ldots,n$$

Thus the sum of all the price elasticities for good i equals the income elasticity for that good. In the special case of all cross elasticities are zero, then $\varepsilon_{ii} = \varepsilon_i = n_i$, or own price elasticity equals income elasticity.

F. AN EQUILIBRIUM SOLUTION

Let equation (16) be the demand function of consumers for the $i^{\underline{th}}$ good. Assume that the level of income I is constant and that all cross elasticities are identically zero. Thus, $\varepsilon_i = n_i$ for all i. Before the disturbance, i.e., before the increase of the price of one primary factor w_1 from r_1 to $r_1 + \Delta r_1$, all goods are cleared at market price, i.e.,

Supply = Demand , or

$$y_i = q_i \quad , \quad i=1,\ldots,n \quad , \tag{18}$$

where y_i refers to the output of the $i^{\underline{th}}$ sector available for final demand and q_i is the total demand for the $i^{\underline{th}}$ good. After the disturbance, y_i changes by the amount Δy_i and q_i by the amount Δq_i, the former being due to the reduction in the amount of input, and the latter due to the price increase of goods. Again, for the special case of two primary input factors and three sectors (k=2,n=3), assume that Δy_3 is given. Thus the equilibrium to exist, it must be that,

$$\Delta q_1 = \Delta y_1 \quad \text{and} \quad \Delta q_2 = \Delta y_2 .$$

Using the elasticities of demand,[1]

$$\frac{\Delta q_1/q_1}{\Delta p_1/p_1} = \varepsilon_1 \quad ,$$

or

(19a) $$\Delta q_1 = \varepsilon_1 q_1 \; \Delta p_1/p_1 \quad , \quad \text{and}$$

(19b) $$\Delta q_2 = \varepsilon_2 q_2 \; \Delta p_2/p_2$$

By equation (18) $q_1 = y_1$ and $q_2 = y_2$, and by equation (15),

$$\Delta p_1 = \Delta r_1 \sum_{j=1}^{3} b_{1j} c_{j1} \quad , \quad \text{and}$$

$$\Delta p_2 = \; r_1 \sum_{j=1}^{n} b_{2j} c_{j2} \quad ,$$

where c_{ij} is the i-j$\underline{\text{th}}$ element of the $(I-A)^{-1}$ matrix. Or

$$\Delta p_1 = \Delta r_1 \; \Sigma_1 \quad , \quad \text{and}$$

$$\Delta p_2 = \Delta r_1 \; \Sigma_2 \quad ,$$

where $\Sigma_1 = \sum_{j=1}^{3} b_{1j} c_{j1}$, and $\Sigma_2 = \sum_{j=1}^{3} b_{2j} c_{j2}$.

---

[1]Elasticity of demand is loosely used here. The elasticity referred to is the arc elasticity which is due to finite changes Δp and Δq. See Ref. [2], p. 18.

Equations (19a) and (19b) become,

(20a) $$\Delta q_1 = \varepsilon_1 \Delta r_1 \, \Sigma_1 \, y_1/p_1$$

(20b) $$\Delta q_2 = \varepsilon_2 \Delta r_1 \, \Sigma_2 \, y_2/p_2 \quad .$$

At the new equilibrium state, it must be that $\Delta q_1 = \Delta y_1$ and $\Delta q_2 = \Delta y_2$. Using equations (14a) and (20a),

$$\varepsilon_1 \Delta r_1 \Sigma_1 y_1/p_1 = g_{11} \Delta w_1 + g_{12} \Delta w_2 \quad .$$

From equation (13a) ,

$$\Delta w_1 = a w_1 \Delta r_1/r_1 \ .$$

Also, the fixed proportion of inputs is,

$$\Delta w_2 = \delta \Delta w_1 \quad .$$

Hence,

$$\varepsilon_1 \Delta r_1 \Sigma_1 y_1/p_1 = g_{11} a w_1 \Delta r_1/r_1 + g_{12} \delta a w_1 \Delta r_1/r_1 \ ,$$

or

(21) $$r_1/p_1 = (a/\varepsilon_1)(w_1/y_1)(g_{11} + \delta g_{12})/\Sigma_1 \quad .$$

Similarly,

$$(22)\qquad r_1/p_2 = (a/\varepsilon_2)(w_1/y_2)(g_{21} + \delta g_{22})/\Sigma_2$$

G. SPECIFIC RESULTS

Equations (21) and (22) show that under the following assumptions:

1. a is the elasticity of demand for w_1 (petroleum),
2. w_2 will change proportionately with w_1 and that the change in w_2 (labor) is assumed to be absorbed by other segments of the economy,
3. ε_i is the elasticity of demand for good i and that $\varepsilon_{ij} = 0$, for all i and j , $i \neq j$, $i,j = 1,\ldots,n$, a disturbance given by a change in price of one of the input factors will result in a return to equilibrium and such equilibrium is attained if equations (21) and (22) are satisfied.

In words, equation (21) states that the ratio between the price of the input factor whose price is changed to the price of good i is equal to the ratio of their elasticities, multiplied by the ratio of their equilibrium quantities, times a constant which is determined from the underlying technology of the input output system. A similar statement can be made for the ratio r_1/p_2 which is called the "cross price ratio." Note however that these results are only restricted to the case of two factor inputs and three sectors. The ratio r_1/p_3 could be arrived at if either Δy_1 or Δy_2 were assumed to be given.

H. A GENERAL RESULT

Assuming that there are two factor inputs and n sectors, a general result could be arrived at. Setting any of the n-2 output goods as exogenous, the change in the output for final demand of any two sectors can be solved. Using equations (11) and (14a),

$$\begin{bmatrix} \Delta y_i \\ \\ \Delta y_j \end{bmatrix} = \begin{bmatrix} g_{1i} & g_{1j} \\ \\ g_{2i} & g_{2j} \end{bmatrix} \begin{bmatrix} \Delta w_1 \\ \\ \Delta w_2 \end{bmatrix} = \begin{bmatrix} g_{1i}\Delta w_1 + g_{1j}\Delta w_2 \\ \\ g_{2i}\Delta w_1 + g_{2j}\Delta w_2 \end{bmatrix} ,$$

where $i,j = 1,\ldots,n$ and $i \neq j$. By similar reasoning as in the $k=2, n=3$ case,

(23) $$r_1/p_i = (a/\varepsilon_i)(w_1/y_i)(g_{1i} + \delta g_{1j})/\Sigma_{1i} \quad , \; i,j=1,\ldots,n,$$

and $i \neq j$. where $\Sigma_{1i} = \sum_{t=1}^{n} b_{1t}c_{ti}$, ε_i the elasticity of demand of the i^{th} good and y_i the output for final demand of the i^{th} sector.

Similarly,

(24) $$r_1/p_j = (a/\varepsilon_j)(w_1/y_j)(g_{2i} + \delta g_{2j})/\Sigma_{2j} \quad , \; i,j=1,\ldots,n,$$

where $\Sigma_{2j} = \sum_{t=1}^{n} b_{2t}c_{tj}$.

Equations (23) and (24) can be expressed in a more compact form:

(23a) $r_1/p_i = (a/\varepsilon_i)(w_1/y_i)K_{1i}$, where $K_{1i} = (g_{1i} + \delta g_{1j})/\Sigma_{1i}$,

and

(24a) $r_1/p_j = (a/\varepsilon_j)(w_1/y_j)K_{2j}$, where $K_{2j} = (g_{2i} + \delta g_{2j})/\Sigma_{2j}$.

For this first approach in specifying or defining a new equilibrium, such state can be attained if equations (23) and (24) are satisfied under the restrictive assumptions previously stated. At equilibrium, the ratio of the factor price to the price of goods produced is independent of the level of production or output as long as the relative amount of input and output does not change. This result is consistent with the homogeneity of degree 1 of production function (constant return to scale) in input-output economics. Moreover, this price ratio r_1/p_i may be changed by varying the values of some of the parameters in the equation. The next section is devoted to exploring this point.

I. MINIMIZATION OF PRICE INCREASE FOR EVERY UNIT INCREASE OF FACTOR PRICE

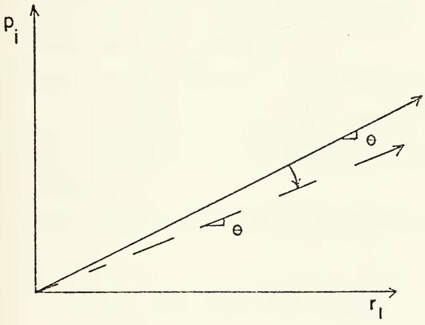
Solving equation (23a) for the price gives,

$$p_i = (\varepsilon_i/a)(y_i/w_1)r_1 K_{1i} .$$

Take the partial derivative of p_i with respect to r_1. This gives the slope, i.e., the rise in price p_i for every unit increase in the factor price r_1. Thus.

$$(25) \qquad \partial p_i/\partial r_1 = (\varepsilon_i/a)(y_i/w_1)\ K_{1i} \equiv \theta \quad .$$

Figure 6 shows this linear relationship between p_i and r_1.



Output Price vs Input Factor Price

Figure 6

The obvious objective is to minimize the price increase in p_i for every unit increase of r_1. This can be done by minimizing the slope θ. Since θ is a function of a number of terms, any of these terms or their combinations may be

changed which could affect the value of θ. Each of these terms will be reviewed as to their meaning and significance so that the necessary sensitivity analysis could be done.

ε_i - the elasticity of demand of consumers for good i. This is considered exogenous since one can hardly control consumers' behavior and changes in taste.

a - The assumed elasticity of demand by producers for petroleum. In reality, producers' demand for input factors are functions of consumer demand for their products. For the present analysis, this is considered a parameter.

y_i - The current output for final demand of industry i. This is equal to the current demand for good i by the consumers. y_i is the equilibrium final output level.

w_1 - The present total quantity of primary input factor 1.

$$K_{1i} = (g_{1i} + \delta g_{1j})/(\sum_{t=1}^{n} b_{1t}c_{ti}) \quad .$$

Substituting K_{1i} into equation (25),

$$\theta = (\varepsilon_i/a)(y_i/w_1)(\sum_{t=1}^{n} b_{1t}c_{ti})/(g_{1i} + \delta g_{1j}) \tag{25a}$$

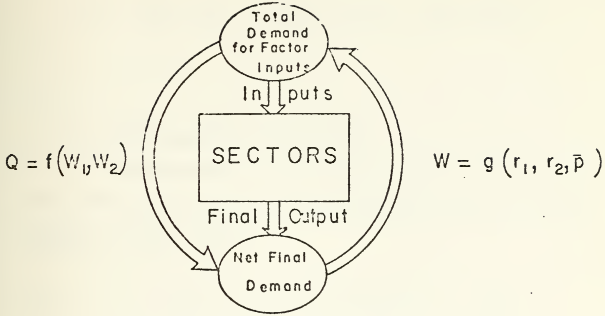
The term b_{1t} , $t=1,\ldots,n$ is the first row of the primary input coefficient matrix. Thus, one way to minimize θ is to minimize b_{1t}. This is equivalent to saying that industry

should have more output for a given input. This is clearly a technological consideration. Moreover, changes in technology will also change the intermediate input coefficients which will affect the $(I-A)^{-1}$ matrix whose elements are the c_{ij}'s. These elements are in the numerator of equation (25a). Reducing these elements will also reduce θ. However, care should be taken in this since some of the elements of $(I-A)^{-1}$ are also embedded in the g_{hj} (elements of $\underline{F}^{-1}$) terms which are in the denominator. Further analysis of this is beyond the scope of this work.

V. THE SECOND APPROACH: STATIC INPUT-OUTPUT ANALYSIS AND AN AGGREGATE PRODUCTION FUNCTION

A. INTRODUCTION

This second approach in defining a new equilibrium state is slightly different from the previous one. This analysis still uses the structural base of an input-output system. However, one bold assumption is made, and that is, the aggregate input and output relationship in the input-output system can be explained by an aggregate production function which satisfies all the requisites of an input-output system, which are: (a) constant return to scale, (b) non-substitutability of factors of production, and (c) long run equilibrium. The Cobb-Douglas production function is shown to satisfy all the above-stated requirements. Figure 7 shows the basic structure of this approach. This figure shows that $Q = f(W_1, W_2)$, where Q is the total output available for final demand, and W_1, W_2 are the total primary input factors of production. The function $W = g(r_1, r_2, \overline{p})$ is the demand function for the vector W by the producers.



Input Output System and an Aggregate Production Function

Figure 7

B. THE COBB-DOUGLAS PRODUCTION FUNCTION

1. <u>Degree of Homogeneity</u>

The production function,

(26) $$q = A\, w_1^{\alpha}\, w_2^{\beta} \quad , \text{ where } \alpha+\beta = 1 \text{ and } A = \text{constant}$$

has a constant return to scale or homogeneity of degree 1. Increasing the input by a factor of n increases the total output also by a factor of n, i.e., if

$$q^o = f(w_1,w_2) = A\ w_1^\alpha\ w_2^\beta \quad , \quad \text{then}$$

$$q^1 = f(nw_1,nw_2) = A(nw_1)^\alpha(nw_2)^\beta = An\ w_1^\alpha\ w_2^\beta$$

$$= nq^o \quad .$$

2. <u>The Marginal Productivity</u>

The Marginal Productivity (MP) of a given input is the partial derivative of the production function with respect to that input.

$$\text{(27a)} \qquad \partial q/\partial w_1 = f_1 = A\alpha w_1^{\alpha-1} w_2^{1-\alpha} \quad , \quad \text{and}$$

$$\text{(27b)} \qquad \partial q/\partial w_2 = f_2 = A(1-\alpha) w_1^\alpha\ w_2^{-\alpha} \quad .$$

The value of the MP of w_1 which is equal to pf_1, p being the price of output q, is the rate at which the entrepreneur's revenue would increase with further application of w_1. The first order condition for profit maximization requires that each input be utilized up to a point at which the value of its MP equals its price. This can be seen by maximizing the profit equation with respect to w_1 and w_2 [3]. Let π be defined as the profit,

$$\pi = \text{TOTAL REVENUE} - \text{TOTAL COST} \ .$$

The total revenue is equal to pq and the total cost is $r_1w_1 + r_2w_2 + b$ where b is the fixed cost. Thus,

$$\pi = pq - r_1w_1 - r_2w_2 - b \quad .$$

Taking partial derivatives with respect to w_1 and w_2 and equating to zero,

$$\partial\pi/\partial w_1 = pf_1 - r_1 = 0 \quad \text{and}$$

$$\partial\pi/\partial w_2 = pf_2 - r_2 = 0 \quad \text{, or}$$

(28a) $$pf_1 = r_1 \quad \text{and}$$

(28b) $$pf_2 = r_2$$

which is as claimed.

The second order conditions for profit maximization require that,

$$\partial^2\pi/\partial w_1^2 = pf_{11} < 0$$

and

$$\partial^2\pi/\partial w_2^2 = pf_{22} < 0 \quad .$$

These conditions imply that further application of either w_1 or w_2 will decrease profit. Furthermore the determinant of the Hessian,

$$\begin{vmatrix} \partial^2\pi/\partial w_1{}^2 & \partial^2\pi/\partial w_1 w_2 \\ \partial^2\pi/\partial w_1 w_2 & \partial^2\pi/\partial w_2{}^2 \end{vmatrix} = p^2 \begin{vmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{vmatrix} > 0$$

implying that profit decreases with further application of both w_1 and w_2.[1]

3. The Expansion Path and Isoquant Lines

Dividing equation (28a) by (28b),

$$(29) \qquad r_1/r_2 = f_1/f_2 \quad .$$

For the Cobb-Douglas function,

$$r_1/r_2 = (\alpha A w_1^{\alpha-1} w_2^{1-\alpha})/((1-\alpha) A w_1 w_2^{-\alpha})$$

$$= \alpha w_2/(1-\alpha) w_1 \quad , \qquad \text{or}$$

$$(30) \qquad (1-\alpha) r_1 w_1 - \alpha r_2 w_2 = 0 \quad .$$

This equation describes a linear expansion path. Figure 8 shows this path.

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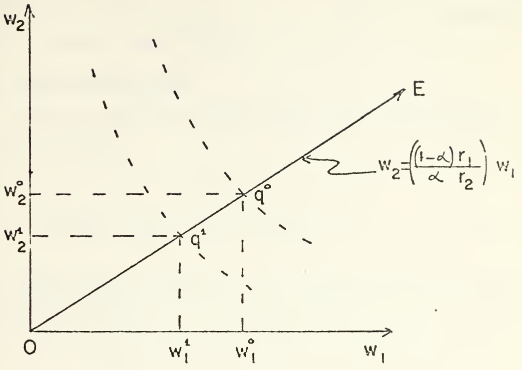
[1]Reference [3], p. 67-68.



Expansion Path and Isoquant Lines

Figure 8

Note that q^o is the output level for inputs w_1^o and w_2^o and q^1 for inputs w_1^1, w_2^1 . Where there is factor substitution , the same output level can be attained by a number of combinations. This set of combinations is given by the dotted lines through q^o and q^1 and are called the isoquant lines (the same quantity level for different input combinations). However, if there were no factor substitution, such as in input-output economics, the levels q^o and q^1 could only be reached by the mix x_1^o and x_2^o , and x_1^1 and x_2^1 , respectively. For example, if the quantity of w_1 is reduced from w_1^o to w_1^1 ,

there will be an excess of $w_2^o - w_2^1$ and the level of output that can be attained given the present technology is q^1. The production function used in this approach is assumed to have no factor substitution.

4. Euler's Equation

Euler's equation states that for a homogeneous function of degree z,

$$(31) \qquad w_1 f_1 + w_2 f_2 = z\, f(w_1, w_2) \quad .$$

For the Cobb-Douglas function, z=1. Substituting in the above equation gives,

$$(31a) \qquad w_1 f_1 + w_2 f_2 = q \quad , \text{ or}$$

$$w_1(\alpha A w_1^{\alpha-1} w_2^{1-\alpha}) + w_2((1-\alpha) A w_1 w_2^{-\alpha}) = q \ , \quad \text{and}$$

$$(32) \qquad \alpha q + (1-\alpha) q = q \quad .$$

This means that if each factor is paid by its marginal product, the total output is distributed between w_1 and w_2 in the proportions α and $(1-\alpha)$ respectively.[1]

Using the first order conditions given by equations (28a) and (28b),

$$pf_1 = r_1 \qquad \text{and} \qquad pf_2 = r_2$$

---

[1]Reference [3], p. 81.

and multiplying through by p gives,

$$w_1(pf_1) + w_2(pf_2) = q \quad .$$

Substituting equations (28a) and (28b),

$$w_1r_1 + w_2r_2 = pq \quad . \tag{33}$$

Thus regardless of the level p, the long run total factor payment equals the revenue.

5. Total Factor Income and the Cobb-Douglas Parameter

It is noteworthy that for the Cobb-Douglas function, the parameter α which is the exponent of w_1 is equal to the total expenditure on petroleum divided by the total expenditure of the energy sectors. That is,

$$q = A\, w_1^{\alpha}\, w_2^{1-\alpha} \quad .$$

Taking partial derivatives with respect to w_1,

$$\partial q/\partial w_1 = A\alpha w_1^{\alpha-1} w_2^{1-\alpha} = \alpha q/w_1 \ .$$

Dividing both sides by q/w_1 ,

$$(\partial q/q)(\partial w_1/w_1) = \alpha \quad . \tag{34}$$

Using equation (28a), $pf_1 = r_1$,

$$p\ \partial q/\partial w_1 = r_1 \quad ,$$

and substituting equation (34),

$$\alpha pq/w_1 = r_1 \quad , \text{ or}$$

$$(35) \qquad \alpha = w_1 r_1/pq \quad ,$$

which is as previously claimed. Thus, for empirical analysis and application of this approach, the parameter α for petroleum could be calculated.

From all of the above discussion, it is seen that the production function of the Cobb-Douglas type satisfies all the requirements of an input-output system.

C. INPUT DEMAND FUNCTION

1. The Demand Equations

The profit-maximizing producers' demand for w_1 and w_2 is dependent on the consumers' underlying demand for his products q. Using the Cobb-Douglas function, form the profit equation and set its partial derivatives equal to zero.

$$\pi = pAw_1^{\alpha}w_2^{\beta} - r_1w_1 - r_2w_2 \qquad , \quad \alpha+\beta < 1$$ [1]

---

[1] If $\alpha+\beta < 1$, the production function is strictly concave for all w_1 and w_2 greater than zero. Ref. [3], p. 61.

$$\partial\pi/\partial w_1 = p\alpha A w_1^{\alpha-1} w_2^{\beta} - r_1 = 0 \quad , \quad \text{and}$$

$$\partial\pi/\partial w_2 = p\beta A w_1^{\alpha} w_2^{\beta-1} - r_2 = 0 \; .$$

Solving for w_1 and w_2, the corresponding demand equations are:[1]

$$w_1 = (\alpha/r_1)^{(1-\beta)/\gamma}(\beta/r_2)^{\beta/\gamma}(Ap)^{1/\gamma} \tag{36a}$$

$$w_2 = (\alpha/r_1)^{\alpha/\gamma}(\beta/r_2)^{(1-\alpha)/\alpha}(Ap)^{1/\gamma} \tag{36b}$$

where $\gamma = 1-\alpha-\beta$.

It can be seen that the demand for w_1 and w_2 decreases as r_1 and r_2 increase, and increases as p increases.

2. <u>The Elasticities of Demand</u>

From equation (13),

$$\varepsilon_1 = (dw_1/w_2)(dr_1/r_1)$$

$$= (r_1/w_1)(\gamma/dr_1)[(\alpha/r_1)^{(1-\beta)/\gamma}(\beta/r_2)^{\beta/\gamma}(Ap)^{1/\gamma}$$

$$= (\beta-1)/\gamma \; .$$

---

[1]Reference [3], p. 69.

Similarly, solving for ε_2, ε_{12} and ε_{21},

$$\varepsilon_2 = (\alpha-1)/\gamma$$

$$\varepsilon_{12} = -\beta/\gamma$$

$$\varepsilon_{21} = -\alpha/\gamma \ ,$$

where ε_{12} and ε_{21} are cross elasticities of demand. Cross elasticity of demand is defined as the ratio of the percentage change in quantity of one factor to the percentage change in price of the other factor. For example, let the price of w_1 increase by Δr_1. Using the elasticities of demand,

$$\varepsilon_1 = (\Delta w_1/w_1)/(\Delta r_1/r_1) \quad .$$

Specifically, $\Delta w_1 = \varepsilon_1 w_1 \Delta r_1/r_1 = w_2(\Delta r_1/r_1)(\beta-1)\gamma \quad ,$

and, $\Delta w_2 = \varepsilon_2 w_2 \Delta r_1/r_1 = -w_2(\alpha/\gamma)(\Delta r_1/r_1) \quad .$

Thus for a change in price Δr_1 of w_1, the total change in quantities demanded is Δw_1 and Δw_2.

D. CHANGE IN PRICE OF AN INPUT FACTOR

1. Change in Quantity of Factor Inputs

Let $Q = f(W_1,W_2)$ be the aggregate production function of all the energy sectors where $W_i = g_i(r_1,r_2,\overline{p})$, i=1,2, and

$\bar{p}$ is the average price of the outputs of all sectors. Let Q be expressed in say energy units such that the unit of outputs of all sectors are all the same. Specifically, using the Cobb-Douglas function,

$$(37) \qquad Q = AW_1^{\alpha}W_2^{\beta} \qquad , \quad \alpha+\beta < 1 ,$$

and W_1 and W_2 are given by equations (36a) and (36b), respectively. Let r_1 change by $\Delta r_1 > 0$. Thus,

$$\Delta W_1 = W_1[(\beta-1)/\gamma](\Delta r_1/r_1) ,$$

$$\Delta W_2 = -W_2[(\alpha/\gamma)](\Delta r_1/r_1) .$$

Recall equations (14a) which is,

$$\Delta y_1 = g_{11}\Delta w_1 + g_{12}\Delta w_2 ,$$

$$\Delta y_2 = g_{21}\Delta w_1 + g_{22}\Delta w_2 ,$$

and use ΔW_1 and ΔW_2 in place of Δw_1 and Δw_2 respectively. Furthermore, assume the same consumer demand equation as given by equation (16), i.e.,

$q_i = h(p_1,p_2,\ldots,p_n,\text{Income})$, and let

$\varepsilon_{ij} = 0$ for $i \neq j$, $i,j = 1,\ldots,n$.

Under a state of equilibrium, supply of good i equals demand for good i. Or,

$$y_i = q_i \quad , \quad i=1,\ldots,n$$

where y_i and q_i are the supply and demand of good i. After the disturbance, y_i changes by an amount of $\triangle y_i$ and q_i by the amount of $\triangle q_i$.

The slight difference between the present approach and the previous one is that, there is only one price $\overline{p}$ for all goods. The disturbance causes the equilibrium price $\overline{p}$ to shift to $\overline{p}+\Delta\overline{p}$. Thus it is necessary to find $\Delta\overline{p}$.

2. The Average Price

Define $\varepsilon_{1\overline{p}}$ as the elasticity of demand for W_1 due to $\overline{p}$. Then,

$$\varepsilon_{1\overline{p}} = (dW_1/W_1)/(d\overline{p}/\overline{p})$$

$$= (W_1/\overline{p})\,[C_1(A\overline{p})^{1/\gamma - 1}(1/\gamma)] = 1/\gamma \quad ,$$

where $\quad C_1 = (\alpha/r_1)^{(1-\beta)/\gamma}(\beta/r_2)^{\beta/\gamma} \quad .$

Also $\quad \varepsilon_{2\overline{p}} = (dW_2/W_2)(d\overline{p}/\overline{p}) = 1/\gamma \;.$

Suppose ΔW_1 has been found as a consequence of the disturbance Δr_1. Then,

$$(\Delta W_1/W_1)(\Delta\bar{p}/\bar{p}) = 1/\gamma \ , \quad \text{and}$$

$$\text{(38)} \qquad \Delta\bar{p} = \Delta W_1 \bar{p}\gamma/W_1 \ .$$

Note that the aggregate production function was only assumed to derive the change in the aggregate level of final output and its average price satisfying,

$$\text{(39)} \qquad r_1 W_1 + r_2 W_2 = Q\bar{p} \ ,$$

where W_1 and W_2 are the total input factors used. The assumption about an aggregate production function to express the relationship between total inputs and total output for fixed demand is valid if:

$$r_1 W_1 + r_2 W_2 = r_1 \sum_{i=1}^{n} w_{1i} + r_2 \sum_{i=1}^{n} w_{2i} \ , \quad \text{or}$$

$$\sum_{i=1}^{n} y_i p_i = Q\bar{p} \ ,$$

where the first equation shows the equality of cost of total expenditures for factors 1 and 2, and the second gives the equality of total revenues for the aggregate function and the input-output system.

3. Estimation of the Changes in Prices of Output Goods

The average price $\overline{p}$ is equal to $(\sum_{i=1}^{n} p_i)/n$.

Using the price equation,

$$P' = (r_1 b_1' + r_2 b_2')(I-A)^{-1} \quad ,$$

$$p_i = \sum_{j=1}^{n} (r_1 b_{1j} + r_2 b_{2j}) c_{ji} \quad .$$

For three sectors,

$$p_i = \sum_{j=1}^{3} (r_1 b_{1j} + r_2 b_{2j}) c_{ji} \quad .$$

Equation (15) states that,

$$\Delta p_i = \Delta r_1 \sum_{j=1}^{3} b_{1j} c_{j1} \quad .$$

Therefore,

$$\Delta \overline{p} = (\Delta r_1/3)[\sum_{j=1}^{3} b_{1j} c_{j1} + \sum_{j=1}^{3} b_{1j} c_{j2} + \sum_{j=1}^{3} b_{1j} c_{j3}] \quad .$$

Thus,

(41a) $$\Delta p_1 = (3\Delta \overline{p}/\Delta r_1) - \sum_{j=1}^{3} b_{1j} c_{j2} - \sum_{j=1}^{3} b_{1j} c_{j3}$$

$$= (3\Delta \overline{p}/\Delta r_1) - \Sigma_{12} - \Sigma_{13}$$

(42b) $\Delta p_2 = (3\Delta\bar{p}/\Delta r_1) - \Sigma_{11} - \Sigma_{13}$

(43c) $\Delta p_3 = (3\Delta\bar{p}/\Delta r_1) - \Sigma_{11} - \Sigma_{12}$,

where, $\Sigma_{1i} = \sum_{j=1}^{3} b_{1j}c_{ji}$.

Hence, given $\Delta\bar{p}$, the changes in prices of good i, i=1,...,n can be estimated using the B and the $(I-A)^{-1}$ matrices of the input-output system.

E. AN EQUILIBRIUM SOLUTION

Using the same procedure as in the first approach equilibrium is attained if the change in supply equals the change in demand, or

$$\Delta q_i = \Delta y_i \quad , \quad i=1,\ldots,n \quad .$$

Again for the special case of k=2,n=3, let $\Delta q_3 = \Delta y_3$. At the current equilibrium point $y_1 = q_1$, $q_2 = q_2$ and $y_3 = q_3$. After the disturbance, the new equilibrium exists if, $\Delta y_1 = \Delta q_1$ and $\Delta y_2 = \Delta q_2$. Using the elasticity of demand and equations (41a) and (41b),

$$\Delta q_1 = \varepsilon_1 q_1(\Delta p_1/p_1) = \varepsilon_1 y_1(\Delta p_1/p_1)$$

$$= (\varepsilon_1 y_1/p_1)[(3\Delta\bar{p}/\Delta r_1) - \Sigma_{12} - \Sigma_{13}] \ ,$$

and

$$\Delta q_2 = \varepsilon_2 q_2(\Delta p_2/p_2) = \varepsilon_2 y_2(\Delta p_2/p_2)$$

$$= (\varepsilon_2 y_2/p_2)[3\Delta\bar{p}/\Delta r_1 - \Sigma_{11} - \Sigma_{13}] \quad .$$

Thus,

$$\Delta q_1 = \Delta y_1 \quad , \quad \text{or}$$

$$(\varepsilon_1 y_1/p_1)[3\Delta\bar{p}/\Delta r_1 - \Sigma_{12} - \Sigma_{13}] = g_{11}\Delta W_1 + g_{12}\Delta W_2 \ .$$

Since $\Delta W_1 = W_1(\beta-1)\Delta r_1/\gamma r_1$, and $\Delta W_2 = -W_2\alpha\Delta r_1/\gamma r_1$, substituting in the above equation,

$$(\varepsilon_1 y_1/p_1)[3\Delta\bar{p}/\Delta r_1 - \Sigma_{12} - \Sigma_{13}] =$$

$$= g_{11}[W_1(\beta-1)\Delta r_1/\gamma r_1] + g_{12}[W_2(-\alpha)\Delta r_1/\gamma r_1]$$

$$= \Delta r_1/\gamma r_1[g_{11}W_2(\beta-1) + g_{12}W_2(-\alpha)] \ .$$

or,

(44a) $$\frac{r_1}{p_1} = (\Delta r_1 G_1)/\{[(3\gamma\varepsilon_1 y_1)(\Delta\bar{p}/\Delta r_1)] - \gamma\varepsilon_1 y_1[\Sigma_{12}+\Sigma_{13}]\}.$$

Similarly,

(44b) $$\frac{r_1}{p_2} = (\Delta r_1 G_2)/\{[(3\gamma\varepsilon_2 y_2)(\Delta\bar{p}/\Delta r_1)] - \gamma\varepsilon_2 y_2[\Sigma_{11}+\Sigma_{13}]\} \quad .$$

Equations (44a) and (44b) could be written in a more compact form. Thus,

$$(45) \qquad r_1/p_1 = (\Delta r_1{}^2 G_1)/(\Delta\bar{p}C_1 - \Delta r_1 R_1) \ , \quad \text{and}$$

$$(46) \qquad r_1/p_2 = (\Delta r_1{}^2 G_2)/(\Delta\bar{p}C_2 - \Delta r_1 R_2) \ , \quad \text{where}$$

$$G_h = g_{h1}W_1(\beta-1) + g_{h2}W_2(-\alpha) \ , \quad h=1,2,$$

$$C_i = 3\gamma\varepsilon_i y_i \ , \quad i=1,2,$$

$$R_1 = -\gamma\varepsilon_1 y_1(\Sigma_{12} + \Sigma_{13}) \ , \quad \text{and}$$

$$R_2 = -\gamma\varepsilon_2 y_2(\Sigma_{11} + \Sigma_{13}) \quad .$$

F. SPECIFIC RESULTS

It can be seen that equations (45) and (46) are similar to equations (21) and (22) respectively except for one important term, and this is the Δr_1 term. This implies that the price ratio of factor input to output is not independent of the change in price. This is a more realistic finding than that made in the first approach. Solving equation (45) for p_1 and taking the derivative with respect to r_1,

$$p_1 = (\Delta r_1{}^2 G_1 r_1)/(\Delta\bar{p}C_1 - \Delta r_1 R_1)$$

$$(37) \qquad \partial p_1/\partial r_1 = (\Delta r_1{}^2 G_1)/(\Delta\bar{p}C_1 - \Delta r_1 R_1) = \phi \quad ,$$

which is the rise in p_1 per unit increase in r_1 (slope). Note that Δr_1 is considered to be exogenous and thus in taking the derivative with respect to r_1, it is considered a constant. Equation (37) reveals that ϕ increases as Δr_1 increases.

A more general relationship similar to the ones in the first approach could be derived for an n-sector and two factor input system. This will not be pursued though in this study. For this second approach, to minimize the price of p_i for every unit price increase of r_1, a similar procedure could be done as in the first approach. Moreover, ϕ can be decreased by either decreasing Δr_1 or changing the coefficients of the input-output matrix.

VI. THE THIRD APPROACH: THE WALRAS-LEONTIEF GENERAL EQUILIBRIUM SYSTEM

A. INTRODUCTION

This model attempts to find an equilibrium solution taking into account the consequences of a change in price of an input factor, as stated in Section II. By now it can be realized that the two preceding approaches were attempts to bridge the gap between the input-output sub-system and the price valuation sub-system of the Leontief input-output model. The input-output subsystem basically refers to the production process, i.e., the relationship between inputs and outputs. Equation (11) completely summarizes this relationship, i.e.,

$$W = B(I-A)^{-1}Y \quad , \quad \text{where} \quad (I-A)^{-1}Y = X \quad .$$

The prices of the input factors as well as the output goods are separately determined. Thus there is the so-called price-valuation sub-system given by equation (7a), i.e.,

$$P' = (r_1 b_1' + r_2 b_2')(I-A)^{-1} \quad , \quad \text{for two input factors.}$$

Equations (7a) and (11) are entirely independent from each other. In fact this separability together with the assumption of fixed technological coefficients are the bases for simplicity of the Leontief input-output model [4]. In the

input-output sub-system, the final demands are usually set as variable constants or parameters. However, if the final demands are not parameters but variables depending on prices of both the output goods and the primary factors, then the model loses its simplicity. In fact a change in the price of a primary factor gives rise to various repercussions on prices of goods which in turn induce changes in final demands and hence outputs.[1]

B. STABILITY OF EQUILIBRIUM - WALRASIAN CONDITION

A state of equilibrium is stable if a disturbance results in a return to equilibrium. It is unstable if it does not. The Walrasian stability condition is based on the assumption that buyers tend to raise their bids if excess demand is positive and sellers tend to lower their prices if it is negative.[2]

Let $S = f(p)$ and $D = g(p)$ where S and D are quantities supplied and demanded, respectively, and f and g are their respective functions that relate them to p, which is the price of good q (see Figure 9). The point E where supply equals demand is the equilibrium point. At E, q^o is the quantity demanded and supplied at price p^o. Suppose a change in price

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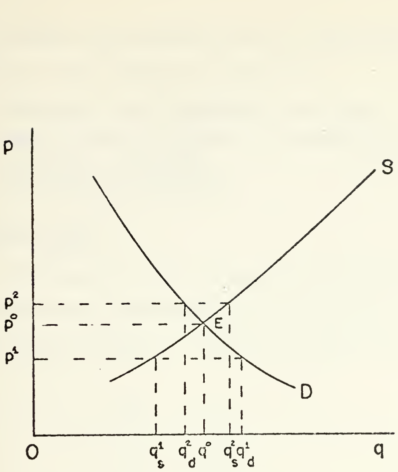
[1]Reference [4], p. 23.

[2]Reference [3], p. 132-136.



Walrasian Stability Condition

Figure 9

occurs and that the new price level is p^1, then the quantity demanded is greater than the quantity supplied, i.e., $q_d^1 > q_s^1$. There is an excess demand of $\Delta q_d = q_d^1 - q_s^1$. Walras stability condition assumes that buyers will raise their bids say to p^2. This will result in a negative excess demand $q_d^2 - q_s^2 < 0$ which makes sellers tend to lower their price. Following the above reasoning, Walrasian stability assumes that the equilibrium point (q^o,p^o) will eventually be reached.

In general the Walras-Leontief equilibrium system consists of the following equations:

1. Household demand functions for goods.
2. Supply-Demand equations for productive factors
3. Price-Cost equations
4. Input-Output equations.

C. ASSUMPTIONS[1]

1. The matrix A is non-negative and indecomposable. An nxn matrix is indecomposable if by permutations of the same rows and columns, A cannot be transformed to the form,

$$\begin{bmatrix} A_1 & A_2 \\ 0 & A_3 \end{bmatrix}$$

with square sub-matrices A_1 and A_3, on the main diagonal.

2. The matrix A satisfies the Hawkins-Simon condition. This condition requires that the feasible gross-output vector X must be positive. The necessary and sufficient condition for there to be a positive solution for X for equation (9) to produce positive net outputs Y is that all principal minors of the (I-A) matrix be positive.[2]

3. The matrix B is non-negative and that for each h, there is at least one j such that $b_{hj} > 0$, i.e., at least one sector uses the $h^{\underline{th}}$ input.

---

[1]Reference [4], p. 25-31.

[2]Reference [2], p. 164.

4. Each y_i is a single-valued and continuous function of p and r, where $p \geq 0$, $r \geq 0$, and that $(p,r) \neq 0$. For any non-negative p and r such that $r \neq 0$, all y_i's are non-negative, and at least one of them is strictly positive.

5. Each w_h is a single-valued and continuous function of p and r, where $p \geq 0$, $r \geq 0$, and $(p,r) \neq 0$. Moreover, if $r_h = 0$, then $w_h = 0$.

6. Each y_i and w_h is homogeneous of degree zero in variables p and r and the following identity (the Walras Law) is satisfied:

$$p'y(p,r) = r'w(p,r) ,$$

where p' and r' are transposes of vectors of prices, y(p,r) means that y is a function of p and r. The same interpretation holds for w.

D. THE GENERAL EQUATIONS

The general equilibrium system of the Walras-Leontief type is formulated by the following three equations:

$$X = AX + Y(p,r) \tag{40}$$

expressing the equality between the demand and supply of goods,

$$BX = W(p,r) \tag{41}$$

expressing the equality between the quantity of productive factors employed and the quantities offered, and the last equation is the price equation which is similar to equation (7a), i.e.,

$$p' = p'A + r'B \quad . \tag{42}$$

Equations (40) and (41) are just generalized expressions of equations (9) and (10) respectively. Morishima[1] [4] claims that under assumptions 1-6, equations (40)-(42) have at least one solution such that $X > 0$, $p > 0$ and $r > 0$.

E. VALIDATION OF FUNCTIONS WITH THE ASSUMPTIONS

1. Assumptions 1-3, and 4 and 5

Assume that matrices A and B satisfy the requirements of Assumptions (1), (2), and (3). For simplicity of illustration, let there be two sectors and two primary factors (n=2, k=2). The equation for fixed demand is,

$$Y = (I-A)X \quad , \quad \text{and}$$

$$y_1 = a_{11}x_1 + a_{12}x_2$$

$$y_2 = a_{21}x_1 + a_{22}x_2 \quad .$$

---

[1]See p. 26-27.

Also, $$x_1 = A_1 w_1^{\alpha_1} w_2^{\beta_1} \text{ , and}$$

$$x_2 = A_2 w_1^{\alpha_2} w_2^{\beta_2} \quad .$$

Furthermore, equations (36a) and (36b) give,

$$w_1^1 = (\alpha_1/r_1)^{(1-\beta_1)/\gamma_1} (\beta_1/r_2)^{\beta_1/\gamma_1} (A_1 p_1)^{1/\gamma_1}$$

$$w_1^2 = (\alpha_2/r_1)^{(1-\beta_2)/\gamma_2} (\beta_2/r_2)^{\beta_2/\gamma_2} (A_2 p_2)^{1/\gamma_2} ,$$

and

$$w_2^1 = (\alpha_1/r_1)^{\alpha_1/\gamma_1} (\beta_1/r_2)^{(1-\alpha_1)/\gamma_1} (A_1 p_1)^{1/\gamma_1}$$

$$w_2^2 = (\alpha_2/r_1)^{\alpha_2/\gamma_2} (\beta_2/r_2)^{(1-\alpha_2)/\gamma_2} (A_2 p_2)^{1/\gamma_2} .$$

The superscript on w refers to the industry demand. Under assumptions (4) and (5), it is clear that $p \geq 0$, $r_1 > 0$ and $r_2 > 0$ for w and y to be defined.

Define $w = g(r_1, r_2, p)$ and S is the subset of R^3, the 3-dimensional Euclidean space. Thus[1] $S \subset R^3$ and g is the function that maps S to $J \subset R^1$, i.e.,

$$S \xrightarrow{g} J \quad .$$

[1]The symbol $\subset$ is read "sub-set of".

Define a point $T^o = (r_1^o, r_2^o, p^o)$ in S such that $g(T^o) = J^o$. g is continuous at T^o in S if for an $\varepsilon > 0$, there is a $\delta > 0$ with the property[1] that

$$|g(T^1) - g(T^o)| < \varepsilon$$

whenever T^1 is in S and $|T^1 - T^o|$ lies within the ball with radius δ centered at T^o. For the function $w = g(r_1, r_2, p)$, S is the set of positive values of r_1 and r_2 and non-negative value of p. Take any point $T^o = (r_1^o, r_2^o, p^o)$ in S,

$$\text{(a)} \qquad g(T^o) = (\alpha/r_1^o)^{\alpha/\gamma} \, (\beta/r_2^o)^{(1-\alpha)/\gamma} \, (Ap^o)^{1/\gamma} \quad .$$

Also, let point $T^1 = (r_1^1, r_2^1, p^1)$ such that T^1 lies within the ball with radius δ. Hence,

$$\text{(b)} \qquad |g(T^1) - g(T^o)| = [(\alpha/r_1^1)^{\alpha/\gamma} \, (\beta/r_2^2)^{(1-\alpha)/\gamma} \, (Ap^1)^{1/\gamma}]$$

$$- \; [(\alpha/r_1^o)^{\alpha/\gamma} \, (\beta/r_2^o)^{(1-\alpha)/\gamma} \, (Ap^o)^{1/\gamma}]$$

$$= (\alpha)^{\alpha/\gamma}(\beta)^{(1-\alpha)/\gamma}(A)^{1/\gamma}[(r_1^1)^{-\alpha/\gamma}(r_2^1)^{(\alpha-1)/\gamma}(p^1)^{1/\gamma}$$

$$- \; (r_1^o)^{-\alpha/\gamma}(r_2^o)^{(\alpha-1)/\gamma}(p^o)^{1/\gamma}] .$$

---

[1]The values of ε and δ are assumed to be close to zero.

As T' approaches T^o, or taking the limit of $g(T^1)$ as T^1 approaches T^o, i.e.,

$$\lim_{T^1 \to T^o} g(T^1) ,$$

the second term (in bracket) of (b) approaches zero since by definition if T^1 approaches T^o, i.e., $T^1 \to T^o$, then $r_1^2 \to r_1^o$, $r_2^1 \to r_2^o$, and $p^1 \to p^o$. Thus for a given $\varepsilon > 0$, $|g(T^1)-g(T^o)| < \varepsilon$ [5]. This is a non-rigorous proof of the continuity of $w = g(r_1,r_2,p)$. It is clear though that $w(r_1,r_2,p)$ is continuous over the region $r_1 > 0$, $r_2 > 0$, and $p \geq 0$. Furthermore, it is assumed that $w = 0$ for $r_1 = 0 = r_2$. w is assumed to be non-negative and hence for the purpose of the present analysis, single-valued and Assumption (5) is satisfied.

Now,

$$X = K\, w_1^{\alpha}\, w_2^{\beta} \quad .$$

Substituting $w_i = g_i(r_1,r_2,p)$, i=1,2, gives,

$$\text{(c)} \qquad X = K[(\alpha/r_1)^{(1-\beta)/\gamma}(\beta/r_2)^{\beta/\gamma}(Ap)^{1/\gamma}]^{\alpha}$$

$$\cdot[(\alpha/r_1)^{\alpha/\gamma}(\beta/r_2)^{(1-\alpha)/\gamma}(Ap)^{1/\gamma}]^{\beta}$$

$$= K[(\alpha/r_1)^{\alpha/\gamma}(\beta/r_2)^{\beta/\gamma}(Ap)^{(\alpha+\beta)/\gamma}] \ .$$

This function is similar to (a) and hence $X(w_1,w_2)$ is continuous over the region $r_1 > 0$, $r_2 > 0$ and $p \geq 0$.

Finally,

$$y_1 = a_{11}x_1 + a_{12}x_2 \quad \text{and} \quad y_2 = a_{21}x_1 + a_{22}x_2 .$$

A theorem in mathematical analysis states that if two functions are continuous at a point T^o in the domain S, then their sum is continuous at T^o. Thus y is continuous.[1] Furthermore, it is intuitively clear that y is also single-valued, and Assumption (5) is satisfied.

2. Assumption 6

The next and last assumption to be verified is the homogeneity of degree zero of y and w in p and r, and Walras' Law. It has been shown for a specific case (see equation (33)) that this is true and the validity of Walras' Law is assumed.

Let

$$y = ax_1 + bx_2 = H(x_1,x_2) \quad , \quad \text{or,}$$

$$y = aA_1 w_1^{\alpha_1} w_2^{\beta_1} + bA_2 w_1^{\alpha_2} w_2^{\beta_2} = H[f(w_1,w_2)] .$$

---

[1]Reference [5], p. 91.

Substituting $w = g(r_1, r_2, p)$ gives,

$$y = H\{f[g(r_1, r_2, p)]\} \quad .$$

Substituting the values of w_1^1, w_1^2, w_2^1 and w_2^2,

$$y = aA_1[(\alpha_1/r_1)^{1/\gamma_1}(\beta_1/r_2)^{1/\gamma_1}(A_1p_1)^{(\alpha_1+\beta_1)/\gamma_1}]$$

$$+ bA_2[(\alpha_2/r_1)^{1/\gamma_2}(\beta_2/r_2)^{1/\gamma_2}(A_2p_2)^{(\alpha_2+\beta_2)/\gamma_2}] \; .$$

Now, let $y^1 = H[nr_1, nr_2, np]$. Thus,

$$y^1 = aA_1[C_1(1/t)^{\alpha_1/\gamma_1}(1/t)^{\beta_1/\gamma_1}(t)^{(\alpha_1+\beta_1)/\gamma_1}]$$

$$+ bA_2[C_2(1/t)^{\alpha_2/\gamma_2}(1/t)^{\beta_2/\gamma_2}(t)^{(\alpha_2+\beta_2)/\gamma_2}]$$

$$= aA_1C_1 + bA_2C_2 \quad , \quad \text{where} \quad C_1 = x_1 \quad \text{and} \quad C_2 = x_2$$

F. AN EQUILIBRIUM SOLUTION EXISTS

Hence, y is homogeneous of degree zero. Similarly, it can be shown that w_h is also homogeneous of degree zero. Thus, Assumption (6) is satisfied and at least one solution exists for equations (40), (41) and (42) such that $x > 0$, $p > 0$ and $r > 0$. A solution for equations (40), (41) and (42) using the appropriate equations in approach two will not be attempted here due to the complexity in calculation even just

for two sectors (n=2) and two primary input factors (k=2). Suffice it to say that the aggregate production function used in approach two together with the derived input demand functions (36a,36b) has an equilibrium solution in the framework of the Walras-Leontief general equilibrium system.

VII. CONCLUSION

This study has attempted to formulate three methods of approach in analyzing the impact of a price change of primary input factor in a segment of an economy using input-output analysis and market mechanisms. This study has arrived at the following conclusions:

1. The final equations arrived at in the first approach (equations 23, 24) are useful in making policy decisions relative to the change in price of a factor input. The direct relationship between the price of an input factor and the price of the $i^{\underline{th}}$ good is established. Aside from being able to minimize price increase of the $i^{\underline{th}}$ good by varying the matrix coefficients, this equation could be further used by looking closely at the other terms which were considered parameters. For example, the effects of changes in elasticities of demand for input factor and also those of the consumers' demand could be analyzed. The value of these results is that a number of important factors that affect market equilibrium are embedded in the equations.

2. The use of the Cobb-Douglas aggregate production function to determine equilibrium output level and level of demand for input factor is another simple way of analyzing the impact of a change in price of a factor input in a segment of an economy. This function satisfies all the requirements of input-output economics. Furthermore, its

practical application can be realized since α which is the exponent of w_1 (petroleum) can be calculated. This parameter could be assumed to be the ratio of the total petroleum expenditure to the total expenditure of all the energy sectors. The final equations (equations 44a, 44b) arrived at which define equilibrium have the price change in input factor included. This is a more useful result than the ones arrived at in the first approach, at the cost though of a set of more complicated equations.

3. The third approach has shown that to incorporate all the equations which includes the consumer demand functions, the demand-supply equations for productive factors, the price-cost equations, and input-output equations, in equilibrium analysis becomes very difficult to mathematically follow. Nevertheless, it has been shown that the equations used in the second approach have an equilibrium solution which validates the results reached in the second approach.

4. Finally, it should be noted that all three approaches were geared towards defining or specifying a new equilibrium state. The rationale behind this is that policy measures taken relative to an economic disturbance should be based or founded on the premise that such measures taken will result in equilibrium at the minimum price increases of output goods. Since this study was motivated by the increase in petroleum prices, the analyses made were geared towards using petroleum as a primary input factor into the energy sectors of an economy. Given that these sectors which

consume a significant portion of petroleum input could be isolated, and that an input-output system be established, i.e., input coefficients, both primary and intermediate, can be computed, then the approaches made in this study will find application in such a situation.

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[1]The succeeding references were not directly referred to but considered very informative in the course of the study.

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Equilibrium analysis of effects of a price change of an input factor in the context of input-output system.

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